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Anomalous Transmission in a Bent Germanium Crystal

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The anomalous transmission of Cu $K\alpha$ radiation through a dislocation-free germanium crystal has been studied by means of the divergent beam method and photographic recording. Elastic bending of the crystal causes a change in the transmission pattern, in full agreement with theoretical calculations of Polder & Penning (1964).

Introduction

Anomalous transmission of X-rays through bent germanium crystals has been studied by Hunter (1958) and Okkerse (1962). In these experiments the change of intensity of a parallel X-ray beam was measured for one special orientation of the crystal and one special way of bending. We thought it worth while to study the intensity distribution of the anomalously transmitted X-ray energy for a variety of directions of incidence of the beam and the change of that distribution caused by bending the crystal in several ways. For this purpose it seemed appropriate to use a beam diverging from a point source and to record the transmitted intensity on a photographic film. In this way we have observed that bending a germanium crystal produces interesting and pronounced effects on the recorded intensity distribution. In a paper by Penning & Polder (1961) the theoretical treatment of the effect of strains on anomalous transmission was given. In that paper the theory was applied to calculate the effects occurring in the experiments of Hunter and Okkerse. The strain pattern was evaluated with the use of an isotropic elastic tensor. The experimental observations reported in the present paper, however, can only be explained if the elastic anisotropy of the germanium crystals is taken into account. In a second theoretical paper by Polder & Penning (1964) this has been done. The pronounced effects predicted by the theory are in exact agreement with our observations. In the description and explanation of the experiments we will use the same notation as Polder & Penning.

Experimental

X-ray tube

The divergent X-ray beam was obtained from a vertically placed end-anode X-ray tube. A schematic drawing of the tube is given in Fig. 1. An electron beam emitted from the electron gun is accelerated by 18 kV to the end window of the tube, which acts as a target. This window consists of a 25 μ copper foil, covered on the outside with 25 μ nickel. An external electromagnetic lens focuses the electron beam to a focal spot of about 40 μ diameter. The X-ray beam energing from this spot mainly consists of Cu $K\alpha$ radiation with a divergence of about 180°.

Specimen

The crystal was a dislocation-free germanium disc 15 mm in diameter and 1·5 mm thick. The upper and lower faces of this disc were ground parallel to the (111) plane (deviation less than 0·1°). After grinding,

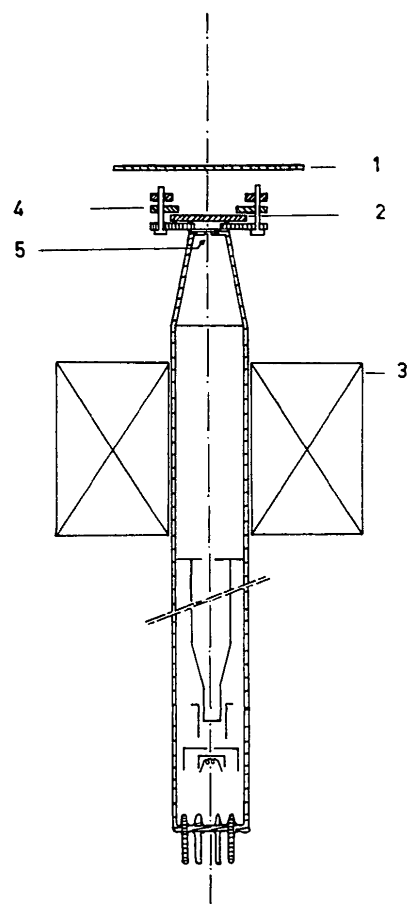

Fig. 1. Schematic drawing of the X-ray tube, showing the crystal in the bending apparatus and the position of the photographic film. 1. Photographic film. 2. Specimen. 3. Electromagnetic lens. 4. Bending apparatus. 5. Point focus.

the crystal was treated in CP-4 solution* and thoroughly rinsed in distilled water.

This crystal, placed between the knife edges of a bending fixture, was put on top of the X-ray tube, with the lower face at a distance of 2 mm from the target. A photographic film parallel to the disc at a distance of 15 mm recorded the X-ray pattern (Fig. 1). The X-ray tube was operated with a current of about 100 μA. The exposure time for the 1·5 mm thick crystal was about 2 hours.

Description of the results

A cone of mainly Cu $K\alpha$ radiation strikes the (111) lower surface of the germanium crystal. Only those X-rays satisfying the Bragg equation for the vertical and oblique {220} planes will be anomalously transmitted through a crystal of this thickness. The anomalous transmission coefficient for other planes and the normal transmission coefficient for other directions are too small to allow measurable transmission through the crystal. Let us consider the anomalous transmission through a vertical (110) plane, for instance the reflexions $20\bar{2}$ and $\bar{2}02$. Fig. 2 gives the section of the X-ray path in a plane through the [111] axis, normal to the $(10\bar{1})$ plane, and containing

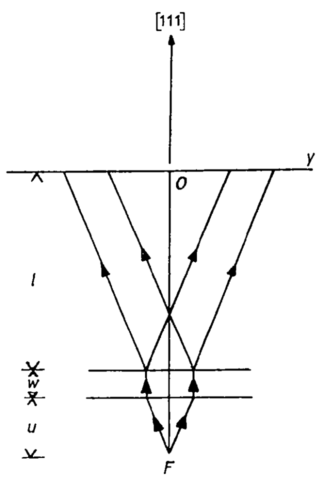

Fig. 2. Representation of the X-ray path in a section through the [111] axis and normal to the $(10\bar{1})$ plane.

the focus F. X-rays from the focus F, incident on the crystal at an angle θ ($\equiv \theta_{220}$) are anomalously transmitted through the crystal in the [111] direction. At the upper crystal surface this beam splits into two beams. One is the transmitted beam in the incident direction (θ) and the other is in the reflected direction ($-\theta$). We define a coordinate system in the photographic film with the geometrical projection of the X-ray source as the origin, the X axis parallel to the $(10\bar{1})$ plane and the Y axis perpendicular to it.

The points where the X-ray beams strike the film have the coordinates:

$$X=0,\ \ Y=\pm(u+l)\tan\theta$$
for the transmitted beam

and

$$X=0,\ \ Y=\mp(u-l)\tan\theta$$
for the reflected beam

where u is the distance between X-ray source and crystal and l the distance between crystal and photographic film. For germanium and Cu $K\alpha$ radiation $\theta_{220}=22°\,39'$ and in our experimental set up $u=2$ mm and $l=15$ mm. The distance between the two outer points on the photograph will be

$$2\times17\tan 22°\,39' = 14{\cdot}1\text{ mm (transmitted beams)}$$

and for the inner points

$$2\times13\tan 22°\,39' = 10{\cdot}8\text{ mm (reflected beams)}.$$

By action of the threefold axis, *i.e.* anomalous transmission through $(10\bar{1})$, $(\bar{1}01)$, $(01\bar{1})$, $(0\bar{1}1)$, $(\bar{1}10)$ and $(1\bar{1}0)$ planes, we get six pairs of points, in a hexagonal symmetry.

If $u=0$, *i.e.* with the crystal direct on the target, each pair of two points will coincide.

We now consider the surface of the cone of X-rays satisfying the Bragg equation for the $(20\bar{2})$ and $(\bar{2}02)$ reflecting planes.

This cone, of which we consider only one half, the $20\bar{2}$ reflexion, strikes the crystal surface as a hyperbola. Each ray from the focus to the hyperbola will make an angle θ with the reflecting $(20\bar{2})$ plane. From each point of the hyperbola the X-ray will travel into the crystal along the $(10\bar{1})$ plane. The angle between this direction of propagation and the [111] axis is called β. In accordance with the definition of β as given by Polder & Penning (1964), the sign of β is positive for our choice of indices, if the angle between the direction of propagation and [101] is smaller than the angle between [111] and [101].

At the upper crystal surface the beam will split into a transmitted and reflected direction, making angles θ and $-\theta$ with the $(10\bar{1})$ planes, and will strike the photographic plate as curved lines. To describe these curves we take in the photographic film the same coordinate system (X, Y) as defined above.

The projection of the path of an arbitrary X-ray, from the point source to the photographic plate on the $(10\bar{1})$ plane, is a line at an angle β with the [111] axis (Fig. 3).

Inside the crystal the beam propagates along this line; outside the crystal the beams make an angle θ and $-\theta$ with it. The X coordinate of the point where an arbitrary X-ray strikes the photographic plate will be given by

* CP-4 is an etch consisting of 15 ml glacial acetic acid, 3 drops bromine, 25 ml 60% nitric acid and 12 ml 50% hydrofluoric acid.

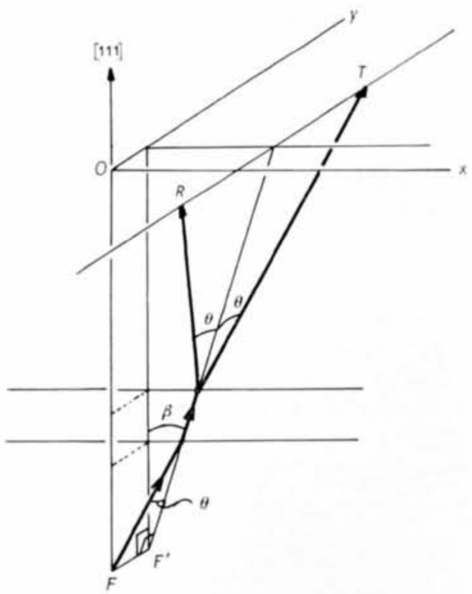

Fig. 3. Representation of the path of an arbitrary X-ray beam anomalously transmitted and reflected by the $(10\bar{1})$ plane.

(1) $X = (u+w+l)\tan\beta$ where w is the crystal thickness.

The Y coordinate will be

(2) $Y = \dfrac{(u+l)}{\cos\beta}\tan\theta$ for the transmitted beam.

(3) $Y = +\dfrac{(u-l)}{\cos\beta}\tan\theta$ for the reflected beam.

Anomalous transmission along the opposite $(\bar{1}01)$ plane will give

(4) $X = (u+w+l)\tan\beta$, and

(5) $Y = -\dfrac{(u+l)}{\cos\beta}\tan\theta$ for the transmitted and

(6) $Y = -\dfrac{(u-l)}{\cos\beta}\tan\theta$ for the reflected beam.

Eliminating β from these equations gives

$$\frac{X^2}{(u+w+l)^2} = -1 + Y^2\frac{\text{cotan}^2\,\theta}{(u+l)^2}$$

for the transmitted beams, the outer curves;

$$\frac{X^2}{(u+w+l)^2} = -1 + Y^2\frac{\text{cotan}^2\,\theta}{(u-l)^2}$$

for the reflected beams, the inner curves.

These equations describe two hyperbolae. By action of the threefold axis we shall find three sets of these hyperbolae in sixfold symmetry. Fig. 4(*a*) gives the photographic recording of these lines.

In this picture we see, besides the six pairs of convex lines in sixfold symmetry, three concave lines in a threefold symmetry. These latter lines are due to the anomalous transmission through the oblique

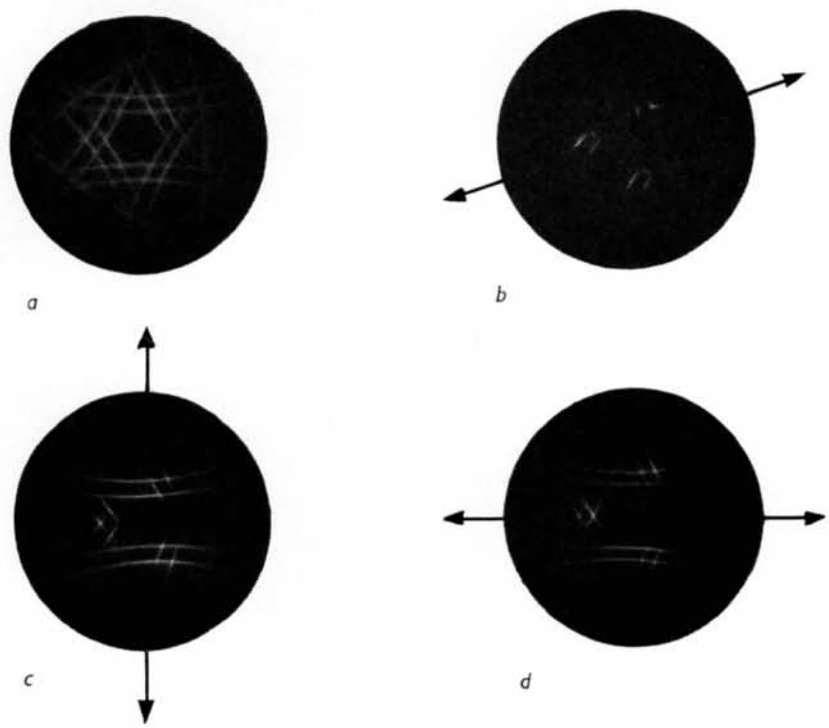

Fig. 4. (*a*) Anomalous transmission pattern of the unbent germanium crystal. (*b*), (*c*), and (*d*) Patterns of the bent crystal. Arrows indicate the direction of bending. In all photographs the crystal has the same orientation.

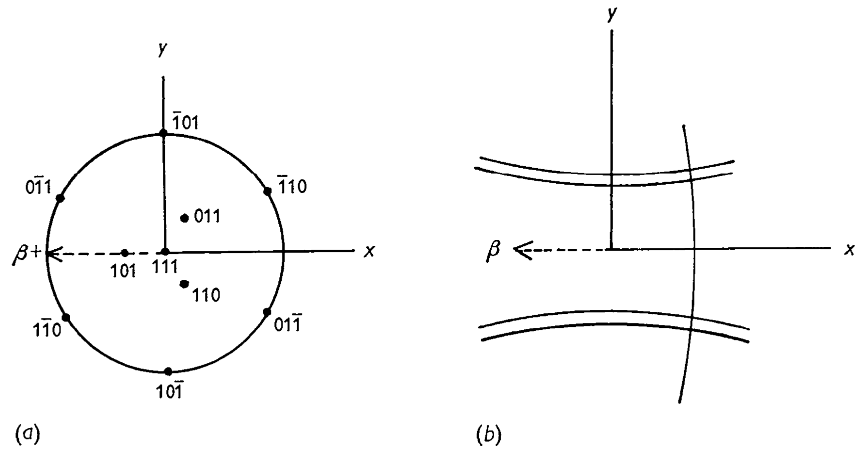

Fig. 5. (*a*) The relative orientation of the X, Y coordinate system, the sign of the angle β, and the crystal axes. The latter given by the stereographic [111] projection. (*b*) Calculated anomalous transmission pattern, for the $20\bar{2}$, $\bar{2}02$ and 202 reflexions.

(110), (011) and (101) planes. This threefold pattern now enables one to determine directly in the photograph the positive direction of β.

Fig. 5(*a*) gives the stereographic [111] projection of the vertical $(\bar{1}10)$ planes and the oblique (101); (110); (011) planes. The positive direction of β for the $(10\bar{1})$ reflecting plane and the coordinate system in our photographic film are also indicated. Careful inspection of the geometric situation shows that the trace of the transmitted beam through the (101) plane arrives at the position indicated in Fig. 5(*b*), where the positive direction of β and the X, Y coordinate system also are indicated. Operation of the threefold [111] axis gives us Fig. 4(*a*).

Now consider what will happen when the crystal is bent in an arbitrary way, *i.e.* in such a way that none of the $(10\bar{1})$, $(\bar{1}10)$ or $(0\bar{1}\bar{1})$ planes is parallel or perpendicular to the direction of bending. According to the calculations of Polder & Penning (1964), anomalous transmission will survive only for rays with a β value given by

$$\tan\beta = -0{\cdot}256\,.$$

Inserting this value in equations (1)–(6) gives us one value for X and two for Y, each for the transmitted and reflected beams, with

$$Y/X = \pm 1{\cdot}48 \text{ for transmission}$$

and

$$Y/X = \pm 1{\cdot}22 \text{ for reflection.}$$

Since arctan $1{\cdot}48 = 56°$, and arctan $1{\cdot}22 = 51°$, surviving intensity will occur not far from the intersection of different pairs of hyperbolae.

Fig. 6 gives a representation of the hyperbolae for transmitted and reflected beams, and the regions where anomalous transmission is calculated to survive bending.

The photographic picture of the anomalous transmission of a bent crystal is shown in Fig. 4(*b*), which is in excellent agreement with the calculated pattern.

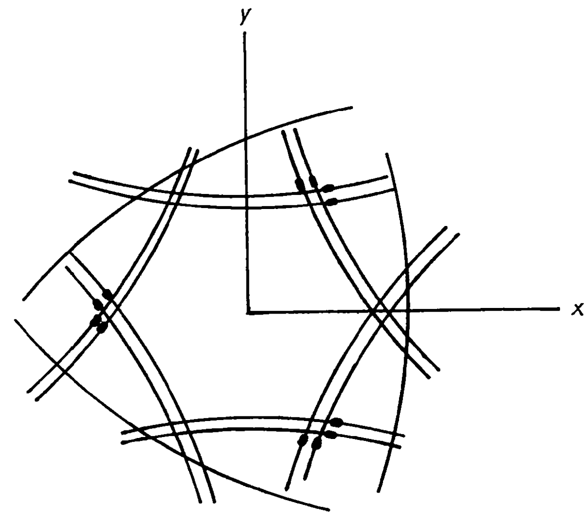

Fig. 6. Calculated anomalous transmission pattern, for an arbitrary bent [111] crystal. Dots give the regions where intensity will survive bending. To be compared with Fig. 4(*a*) and (*b*).

If the direction of stress is either perpendicular or parallel to the $(10\bar{1})$ plane, it follows from formula (19) of Polder & Penning (1964) that no reduction of intensity will be observed for that plane. The other reflexions will follow the normal behaviour. This is shown in Fig. 4(*c*) for the stress pattern perpendicular and in Fig. 4(*d*) for the stress pattern parallel to the $(10\bar{1})$ plane.

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